One-loop Factorization for Inclusive Hadron Production in pA Collisions in the Saturation Formalism

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Abstract

We demonstrate the QCD factorization for inclusive hadron production in pA collisions in the saturation formalism at one-loop order, with explicit calculation of both real and virtual gluon radiation diagrams. The collinear divergences associated with the incoming parton distribution of the nucleon and the outgoing fragmentation function of the final state hadron, as well as the rapidity divergence with small-x dipole gluon distribution of the nucleus are factorized into the splittings of the associated parton distribution and fragmentation functions and the energy evolution of the dipole gluon distribution function. The hard coefficient function is evaluated at one-loop order, and contains no divergence.

1. Introduction. Gluon saturation at small-x (the longitudinal momentum fraction) in nucleon and nucleus plays a crucial role in the description of high energy hadronic scattering [1–3]. It has been applied to a wide range of processes which are relevant for the current and future experiments at RHIC and LHC [4]. Among these physics, suppression of single inclusive hadron production in the forward direction of dAu collisions at RHIC has been regarded as one of the most important evidences for the on-set of gluon saturation at small-x in a large nucleus [5]. In particular, the dipole gluon distribution applied to hadron production in nucleon-nucleus scattering is the same as that in the description of inclusive deep inelastic scattering structure function at small-x [6]. The duality between these two processes, and the universality of the gluon saturation is important to understand the strong interaction dynamics in the dense medium [7].

However, the experimental data are so far interpreted in the leading order calculation in the saturation formalism [8]. In order to solidate the above conclusion, we need to go beyond the leading order Born approximation, and evaluate higher order corrections, which has not yet been done. A next-to-leading order calculation is highly anticipated in the small-x formalism, to demonstrate the QCD factorization which can be applied to a wide range of high energy processes. It has been one of the most important quests in the last few years [9, 10]. Recent studies on the two-particle production in pA collisions have also emphasized the importance of the factorization and universality issues involved in the hard processes, in particular, for small-x physics [11].

In this paper, we will carry out, for the first time, a complete next-to-leading order (NLO) calculation for the single inclusive hadron production in pA collisions in the saturation regime, by employing both the collinear factorization and the high energy small-x factorization techniques. Early efforts have been made in Refs. [12, 13]. Our results will not only provide an important estimate of high order corrections, but also pave the way to build a QCD factorization for hard processes in the saturation formalism [9]. Moreover, this calculation, combining with the associated calculations for inclusive DIS process [10], will help to identify the universality of the dipole gluon distributions in these processes. The method developed here will be very useful to other hard processes in small-x physics as well.

In the process of hadron production in pA collisions,

$$p + A \to h + X \ , \tag{1}$$

a parton from the nucleon (with momentum p) scatters on the nucleus target (with momentum P_A), and fragments into final state hadron with momentum P_h. In the dense medium of the large nucleus and at small-x, the multiple interaction become important, and we have to perform the relevant resummation to make the reliable theoretical calculations. The small-x formalism, the color-glass-condensate or color-dipole formalism, was developed to calculate these hard processes [4]. We are particularly interested in the scattering process with a dilute projectile on a dense target such as the process of (1), where the parton from the incoming nucleon can be treated as the normal parton distribution and it fragments to the final state hadron after multiple interaction with the nucleus target. According to our following calculations, the QCD factorization formalism for this process reads as,

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \sum_a \int \frac{dz}{z^2}\frac{dx}{x}\xi x f_a(x,\mu) D_{h/c}(z,\mu) \int [dx_\perp] S^Y_{a,c}([x_\perp]) \mathcal{H}_{a\to c}(\alpha_s, \xi, [x_\perp]\mu) \ , \tag{2}$$

where $\xi = \tau/xz$ with $\tau = p_\perp e^y/\sqrt{s}$, y and $p_\perp$ the rapidity and transverse momentum for the final state hadron and s the total center of mass energy square $s = (p + P_A)^2$, respectively.

In the above equation, $f_a(x)$ and $D_{h/c}(z)$ represent the collinear parton distribution from the incoming nucleon and fragmentation function for the final state hadron, where x is the momentum fraction of the nucleon carried by the parton a, and z the momentum fraction of parton c carried by the final state hadron h, respectively. The response from the nucleus target is denoted as $S^Y_{a,c}(x_\perp)$ (see the definitions below), depending on the flavor of the incoming and outgoing partons and the gluon rapidity Y associated with the nucleus: $Y \approx \ln(1/x_g)$ with x_g being momentum fraction of nucleus entering the hard process. At the leading order, they are defined as the two-point functions representing the dipole gluon distribution functions in the elementary and adjoint representations for the quark and gluon initialed subprocesses [4], respectively. Higher order corrections will have terms that depend on the correlation functions beyond the simple two-point functions. Because of this reason, the integral $[dx_\perp]$ represents all possible integrals at the particular order. For example, a four-point function (non-linear term of dipole gluon distribution) will contribute to the quark channel at one-loop order. The hard factor $\mathcal{H}_{a\to c}$ describes the partonic scattering amplitude of parton a into a parton c in the dense medium. This hard factor includes all order perturbative corrections, and can be calculated order by order. Although there is no simple $k_\perp$-factorization form beyond leading order formalism [14], we will find that in the coordinate space, the cross section can be written into a nice factorization form as Eq. (2). Besides the explicit dependence on the variables shown in Eq. (2), there are implicit dependences on $p_\perp[x_\perp]$ in the hard coefficients as well.

The above factorization is derived in the high energy limit, i.e., $\sqrt{s} \to \infty$, which is the same limit that the dipole formalism was derived for the inclusive DIS structure function at small-x [15]. Two important variables are introduced to separate different factorizations for the physics involved in this process: the collinear factorization scale μ and the energy evolution rapidity dependence Y. The physics associated with μ follows the normal collinear QCD factorization, whereas the rapidity factorization Y takes into account the small-x factorization. The evolution respect to μ is controlled by the usual DGLAP evolution, whereas that for S^Y_a by the Balitsky-Kovchegov (BK) evolution [16, 17]. In particular, our one-loop calculations will demonstrate the important contribution from this rapidity divergence.

A different formula has been proposed in Ref. [12], where only part of one-loop calculations were taken into account, and the rapidity divergence is not identified and the collinear evolution effects are not complete. In particular, the formula in Ref. [12] was expressed in the leading order form, which does not allow higher order corrections. Recently, it has been realized that the higher order corrections are important for high $p_\perp$ particle production in pA collisions, referred as "inelastic" contribution in Ref. [13]. In this paper, we will demonstrate the factorization formula of Eq. (2), by a complete one-loop calculations, including not only the contributions considered in Refs. [12, 13], but also the virtual diagrams which have not yet been calculated before. We will take the example of quark channel contribution to demonstrate this factorization, and list the results for all other channels in the end. For the quark channel contribution: $qA \to q + X$, the above factorization formula can be explicitly written as

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi xq(x,\mu)D_{h/q}(z,\mu)\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\left\{S^{(2)}(x_\perp,y_\perp)\left[\mathcal{H}^{(0)}_{2qq} + \frac{\alpha_s}{2\pi}\mathcal{H}^{(1)}_{2qq}\right]\right.$$
$$\left. + \int \frac{d^2b_\perp}{(2\pi)^2}S^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}^{(1)}_{4qq}\right\} , \qquad (3)$$

up to one-loop order. The two-point and four-point functions are defined as

$$S^{(2)}(x_\perp, y_\perp) = \frac{1}{N_c}\langle U(x_\perp)U^\dagger(y_\perp)\rangle_Y \tag{4}$$

$$S^{(4)}(x_\perp, b_\perp, y_\perp) = \frac{1}{N_c^2}\langle \mathrm{Tr}[U(x_\perp)U^\dagger(b_\perp)]\mathrm{Tr}[U(b_\perp)U^\dagger(y_\perp)]\rangle_Y \ , \tag{5}$$

with N_c the number of color in QCD, and $U(x_\perp) = \mathcal{P}\exp\left\{ig_S\int_{-\infty}^{+\infty}\mathrm{d}x^+\, T^c A_c^-(x^+, x_\perp)\right\}$ is the Wilson line in the small-x formalism [4] with $A_c^-(x^+, x_\perp)$ being the gluon field solution of the classical Yang-Mills equation. For convenience, we also introduce the momentum space interpretations of the above functions: $\mathcal{F}(k_\perp) = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} S^{(2)}(x_\perp, y_\perp)$, and $\mathcal{G}(k_\perp, k_{1\perp}) = \int \frac{d^2x_\perp d^2y_\perp d^2b_\perp}{(2\pi)^4} e^{-ik_\perp\cdot(x_\perp - b_\perp) - ik_{1\perp}\cdot(b_\perp - y_\perp)} S^{(4)}(x_\perp, b_\perp, y_\perp)$. The above two functions are related as follows: $\mathcal{F}(k_\perp) = \int d^2k_{1\perp}\mathcal{G}(k_\perp, k_{1\perp})$. At the leading order in α_s, there is only two-point function contribution. At the next-to-leading order, a non-linear term of the two-point functions will enter. For the gluon channel, a six-point function will apply. The goal of this paper is to demonstrate the above factorization formalism, and obtain the hard coefficients $\mathcal{H}_{2,4}^{(1)}$. In doing so, we calculate the scattering amplitude squared at one-loop order, and factorize out the divergences associated with the splittings of the quark distribution and fragmentation functions as well as the rapidity divergence from the two-point function. The hard coefficients are Infra-red and Ultra-violet finite.

2. One-loop Calculations. The leading order results have been calculated before, from which we have

$$\mathcal{H}_{2qq}^{(0)} = e^{-ik_\perp\cdot r_\perp}\delta(1-\xi) \ , \tag{6}$$

where $k_\perp = p_\perp/z$ and $r_\perp = x_\perp - y_\perp$. In the following, we will perform the NLO calculations. There are virtual and real gluon radiations. We plot the typical diagrams for them in Fig. 1. After some algebra, we find that the sum of the virtual diagrams,

$$-\frac{\alpha_s}{2\pi^2}\int_0^1 d\xi \frac{1+\xi^2}{1-\xi}\left\{C_F\int d^2q_\perp \mathcal{I}(q_\perp, k_\perp) + \frac{N_c}{2}\int d^2q_\perp d^2k_{g1\perp}\mathcal{J}(q_\perp, k_\perp, k_{g1\perp})\right\} \ , \tag{7}$$

where $C_F = (N_c^2-1)/2N_c$, and $\mathcal{I}$ and $\mathcal{J}$ are defined as

$$\mathcal{I}(q_\perp, k_\perp) = \mathcal{F}(k_\perp)\left[\frac{q_\perp - k_\perp}{(q_\perp - k_\perp)^2} - \frac{q_\perp - \xi k_\perp}{(q_\perp - \xi k_\perp)^2}\right]^2 ,$$

$$\mathcal{J}(q_\perp, k_\perp, k_{g1\perp}) = \left[\mathcal{F}(k_\perp)\delta^{(2)}(k_{g1\perp} - k_\perp) - \mathcal{G}(k_\perp, k_{g1\perp})\right]\frac{2(q_\perp - \xi k_\perp)\cdot(q_\perp - k_{g1\perp})}{(q_\perp - \xi k_\perp)^2(q_\perp - k_{g1\perp})^2} \ .$$

In the above results, we notice that the Ultra-violet divergences cancel out among the virtual diagrams. This can be seen from the large $q_\perp$ behavior of the above integral and the identity relating $\mathcal{G}$ and $\mathcal{F}$. At large $q_\perp$ limit, the gluonic interaction with nucleus target is not affected by the medium effect, and the Ward identity applies, therefore the UV divergence cancels out between the self-energy and vertex diagrams. The real gluon radiation diagrams have been considered before [11, 18]. In order to obtain the single inclusive particle production cross sections, we need to integrate out the phase space of the radiated gluon. In the high energy limit, this integration will collapse the quadrupole into dipole and non-linear term

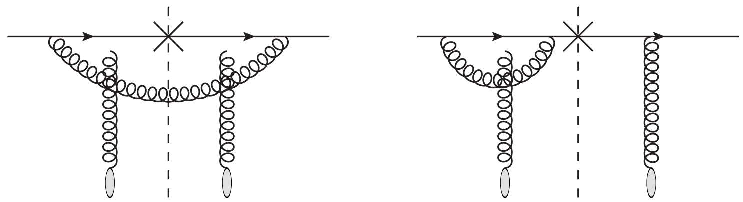

FIG. 1. Typical Feynman diagrams for the real (a) and virtual (b) gluon radiation contributions to the quark production $qA \to q + X$ at one-loop order. The crosses represent the final observed quark, and the blobs in the lower parts of the diagrams represent the multiple interactions with the nucleus at small-x.

of dipoles. At the end of the day, we can cast the real contribution into

$$\frac{\alpha_s}{2\pi^2}\int \frac{dz}{z^2}D(z)\int_{\tau/z}^1 d\xi \frac{1+\xi^2}{1-\xi}xq(x)\left\{C_F\int d^2k_{g\perp}\mathcal{I}(k_\perp,k_{g\perp}) + \frac{N_c}{2}\int d^2k_{g\perp}d^2k_{g1\perp}\mathcal{J}(k_\perp,k_{g\perp},k_{g1\perp})\right\}\ , \tag{8}$$

where $x = \tau/z\xi$.

There are rapidity divergences in the above results (7,8) when $\xi \to 1$. The rapidity divergence appears when the longitudinal momentum of the gluon $k_g^+ = (1-\xi)p^+$ goes to zero, namely when the rapidity of the gluon goes to $-\infty$. It is important to note that the rapidity divergence cancels out for the term proportional to the color-factor C_F. The remainder of the rapidity divergence is proportional to color-factor $N_c/2$, which is exactly the same as that in the BK evolution equation [16, 17]. It is also interesting to note that the rapidity divergence disappears when one integrates over the transverse momentum $k_\perp$ [19].

There are also collinear divergences from both real and virtual diagrams. We use dimensional regularization ($D = 4-2\epsilon$) and follow the $\overline{\text{MS}}$ subtraction. Since the soft gluon radiation has been included into the BK evolution of the unintegrated gluon distribution, there is no soft divergence. For example, for the virtual diagrams we have

$$\frac{2}{4\pi}\left\{C_F\mathcal{F}(k_\perp)\left[-\frac{1}{\epsilon}+\ln\frac{k_\perp^2}{\mu^2}\right]+\left(C_F-\frac{N_c}{2}\right)\mathcal{F}(k_\perp)\ln(1-\xi)^2 + \frac{N_c}{2}\int d^2k_{g1\perp}\mathcal{G}(k_\perp,k_{g1\perp})\ln\frac{(k_{g1\perp}-\xi k_\perp)^2}{k_\perp^2}\right\}\ . \tag{9}$$

The collinear divergence is represented by $1/\epsilon$ in the above equation. The same divergence appears in the real diagram calculations. By summing up both real and virtual contributions, we identify the following divergences,

$$-\frac{\alpha_s N_c}{2\pi^2}\int_0^1\frac{d\xi'}{1-\xi'}\int\frac{d^2b_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}\frac{(x_\perp-y_\perp)^2}{(x_\perp-b_\perp)^2(y_\perp-b_\perp)^2}\left[S^{(2)}(x_\perp,y_\perp)-S^{(4)}(x_\perp,b_\perp,y_\perp)\right] + \frac{\alpha_s C_F}{2\pi}\int_{\tau/z}^1 d\xi\left(-\frac{1}{\epsilon}\right)\left[\mathcal{P}_{qq}(\xi)e^{-ik_\perp\cdot r_\perp}+\mathcal{P}_{qq}(\xi)\frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right]\frac{1}{(2\pi)^2}S^{(2)}(x_\perp,y_\perp)\ , \tag{10}$$

where the splitting kernel is defined as $\mathcal{P}_{qq}(\xi) = \left(\frac{1+\xi^2}{1-\xi}\right)_+$. Obviously, Eq. (10) contains three divergences: rapidity divergence, and two collinear divergences. The rapidity divergence (the first term) can be absorbed into the renormalization of the dipole gluon distribution [9, 10, 16, 17]. By doing so, we introduce the Y dependence in the two-point function, from which the BK evolution can be understood by identifying $dY = d\xi'/(1-\xi')$. The collinear divergences symboled by $1/\epsilon$ in dimensional regularization will be absorbed into the renormalization of the quark distribution and fragmentation functions, following the usual collinear factorization. After subtracting the above divergences, we obtain the hard coefficients

$$\begin{aligned}\mathcal{H}_{2qq}^{(1)} &= C_F \mathcal{P}_{qq}(\xi) \ln \frac{c_0^2}{r_\perp^2 \mu^2} \left(e^{-ik_\perp \cdot r_\perp} + \frac{1}{\xi^2} e^{-i\frac{k_\perp}{\xi}\cdot r_\perp} \right) - 3C_F \delta(1-\xi) e^{-ik_\perp \cdot r_\perp} \ln \frac{c_0^2}{r_\perp^2 k_\perp^2} \\ &\quad - (2C_F - N_c)\, e^{-ik_\perp\cdot r_\perp} \left[\frac{1+\xi^2}{(1-\xi)_+} \widetilde{I}_{21} - \left(\frac{(1+\xi^2)\ln(1-\xi)^2}{1-\xi} \right)_+ \right], \qquad (11)\\ \mathcal{H}_{4qq}^{(1)} &= -4\pi N_c e^{-ik_\perp\cdot r_\perp} \left\{ e^{-i\frac{1-\xi}{\xi}k_\perp\cdot(x_\perp - b_\perp)} \frac{1+\xi^2}{(1-\xi)_+} \frac{1}{\xi} \frac{x_\perp - b_\perp}{(x_\perp - b_\perp)^2} \cdot \frac{y_\perp - b_\perp}{(y_\perp - b_\perp)^2} - \delta(1-\xi) \right.\\ &\quad \left. \times \int_0^1 d\xi' \frac{1+\xi'^2}{(1-\xi')_+} \left[\frac{e^{-i(1-\xi')k_\perp\cdot(y_\perp - b_\perp)}}{(b_\perp - y_\perp)^2} - \delta^{(2)}(b_\perp - y_\perp) \int d^2 r'_\perp \frac{e^{ik_\perp \cdot r'_\perp}}{r'^2_\perp} \right] \right\}, \qquad (12)\end{aligned}$$

where $c_0 = 2e^{-\gamma_E}$ with γ_E the Euler constant, and

$$\widetilde{I}_{21} = \int \frac{d^2 b_\perp}{\pi} \left\{ e^{-i(1-\xi)k_\perp\cdot b_\perp} \left[\frac{b_\perp \cdot (\xi b_\perp - r_\perp)}{b_\perp^2 (\xi b_\perp - r_\perp)^2} - \frac{1}{b_\perp^2} \right] + e^{-ik_\perp\cdot b_\perp} \frac{1}{b_\perp^2} \right\}. \qquad (13)$$

These hard coefficients do not contain any divergence. The calculations for all other partonic channels follow the same procedure, and the hard coefficients are calculated up to one-loop order.

The above results clearly demonstrate that the factorization of (3) is achieved. The collinear divergences associated with the collinear parton distribution and fragmentation functions are factorized, and so does the rapidity divergence associated with the two-point function of the nucleus. This is a very important step to prove the factorization beyond the leading order in perturbation theory.

Our results also show that we can write down the differential cross section in a factorization form in the coordinate space. The factorization scale dependence in the hard coefficients reflect the usual DGLAP evolutions for the quark distribution and fragmentation functions. It is interesting to note that similar μ dependence (associated with $r_\perp$) has also been found in the transverse momentum resummation formalism derived for the Drell-Yan lepton pair production in Ref. [20]. However, the hard coefficients in our case do not contain double logarithms, and there is no need for the Sudakov resummation for inclusive hadron production in pA collisions.

Great simplification for the above hard coefficients can be found if we take large N_c limit. For example, the last term of Eq. (11) will drop out. This applies to all other quark involved

channels as well. Here, we list these hard coefficients in the large N_c limit,

$$
\begin{aligned}
H_{2qg}^{(0,1)} &= H_{2gq}^{(0,1)} = 0 \ , \quad H_{2qq}^{(1)} = -\frac{3}{2} N_c \delta(1-\xi) e^{-ik_\perp \cdot r_\perp} \ln \frac{c_0^2}{r_\perp^2 k_\perp^2} \ , \\
H_{4qg}^{(1)} &= -4\pi N_c \frac{1+(1-\xi)^2}{\xi^2} \frac{x-y}{(x_\perp - y_\perp)^2} \cdot \frac{b_\perp - y_\perp}{(b_\perp - y_\perp)^2} \mathcal{W}(\frac{k_\perp}{\xi}, k_\perp) \ , \\
H_{4gq}^{(1)} &= -4\pi \frac{\xi^2 + (1-\xi)^2}{\xi} \frac{x_\perp - y_\perp}{(x_\perp - y_\perp)^2} \cdot \frac{b_\perp - y_\perp}{(b_\perp - y_\perp)^2} \mathcal{W}(k_\perp, \frac{k_\perp}{\xi}) \ ,
\end{aligned} \tag{14}
$$

where $\mathcal{W}(k_{1\perp}, k_{2\perp}) = e^{-ik_{1\perp}\cdot(x_\perp - y_\perp) - ik_{2\perp}\cdot(y_\perp - b_\perp)}$, and we have chosen $\mu = c_0/r_\perp$ for the factorization scale to further simplify the above expressions. The complete μ dependence can be re-stored by using the DGLAP evolution for the relevant parton distributions and fragmentation functions. With the McLerran-Venugopalan model for the two-point function [3], we find that the best choice for μ will be in the order of the saturation scale Q_s. For the gluon channel $gA \to gX$, a factorization similar to Eq. (3) was established, where the two-point function in the adjoint representation (at large N_c limit as $S^{(2)}(x,y)S^{(2)}(y,x)$) appears at the LO and NLO, and a non-linear term as $S^{(4)}(x,b,y)S^{(2)}(y,x)$ enters at NLO. The relevant hard coefficients at large N_c limit read as: $H_{2gg}^{(0)} = H_{2qq}^{(0)}$, $H_{2gg}^{(1)} = \frac{22}{9} H_{2qq}^{(1)}$, and

$$
\begin{aligned}
H_{6gg}^{(1)} = -16\pi N_c e^{-ik_\perp \cdot r_\perp} \Bigg\{ & e^{-i\frac{k_\perp}{\xi}\cdot(y-b)} \frac{[1-\xi(1-\xi)]^2}{(1-\xi)_+} \frac{1}{\xi^2} \frac{x_\perp - y_\perp}{(x_\perp - y_\perp)^2} \cdot \frac{b_\perp - y_\perp}{(b_\perp - y_\perp)^2} \\
& - \delta(1-\xi) \int_0^1 d\xi' \left[\frac{\xi'}{(1-\xi')_+} + \frac{1}{2}\xi'(1-\xi') \right] \\
& \times \left[\frac{e^{-i\xi' k_\perp \cdot (y_\perp - b_\perp)}}{(b_\perp - y_\perp)^2} - \delta^{(2)}(b_\perp - y_\perp) \int d^2 r'_\perp \frac{e^{ik_\perp \cdot r'_\perp}}{r'^2_\perp} \right] \Bigg\} \ ,
\end{aligned} \tag{15}
$$

respectively. The quark loop contribution has been calculated but not included here.

The above results can be compared to the complete collinear factorization calculation for inclusive hadron production at large transverse momentum. This corresponds to the situation discussed in Ref. [13]. Another important issue is the running coupling effects (see e.g., Ref. [21]) at one-loop order, which has not been discussed in this paper. We will carry out these studies, together with the phenomenological applications to the RHIC and LHC experiments, in a separate publication.

3. Summary. We have demonstrated the QCD factorization for inclusive hadron production in pA collisions in the saturation formalism. The collinear divergences are shown to be factorized into the splittings of the parton distribution from the incoming nucleon and the fragmentation function for the final state hadron. The rapidity divergence at one-loop order is factorized into the BK evolution for the dipole gluon distribution of the nucleus. The hard coefficients are calculated up to one-loop order. In principle, using these hard coefficients together with the NLO parton distributions and fragmentation functions as well as the NLO small-x evolution equation[22, 23], one can obtain the complete NLO cross section of the inclusive hadron production in pA collisions.

These results are very important, not only for the phenomenological applications to the inclusive hadron production in pA collisions at RHIC and the LHC where active experiments are pursued for the study of saturation physics, but also for theoretically promoting the rigorous developments toward a complete QCD factorization in small-x physics [9]. Our

results will stimulate further applications of the method, in particular, the factorization technique, used in this paper to other hard processes involving big nucleus and small-x gluon distributions, as well as those in hot/dense medium. We expect more exciting developments along this line.

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